TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

TOBB-ETÜ, MATEMATİK BÖLÜMÜ, GÜZ DÖNEMİ 2009-2010
MAT 201, DOĞRUSAL CEBİR, 1. ARASINAV
14 EKİM 2009

Adı Soyadı: No: İMZA:

1.	2.	3.	4.	TOPLAM

NOT: Tam puan almak için yeterli açıklama yapılması gerekmektedir.
Sınav süresi 100 dakikadır. Başarılar.

1. *(25 puan)* Aşağıdaki denklem sisteminin çözümlerini a sayısının aldığı değerlere bağlı olarak irdeleyiniz.

$$\begin{aligned} x - y + 2z &= -1 \\ 3x + y - z &= 4 \\ x - 4y + az &= 0 \\ 2x + 2y - 3z &= a+4 \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 3 & 1 & -1 & 4 \\ 1 & -4 & a & 0 \\ 2 & 2 & -3 & a+4 \end{array}\right] \begin{array}{l} -3r_1+r_2 \to r_2 \\ \longrightarrow \\ -r_1+r_3 \to r_3 \\ -2r_1+r_4 \to r_4 \end{array} \left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 0 & 4 & -7 & 7 \\ 0 & -3 & a-2 & 1 \\ 0 & 4 & -7 & a+6 \end{array}\right]$$

$$r_3+r_2 \to r_2 \longrightarrow \left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 0 & 1 & a-9 & 8 \\ 0 & -3 & a-2 & 1 \\ 0 & 4 & -7 & a+6 \end{array}\right] \begin{array}{l} 3r_2+r_3 \to r_3 \\ \longrightarrow \\ -4r_2+r_4 \to r_4 \end{array} \left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 0 & 1 & a-9 & 8 \\ 0 & 0 & 4a-29 & 25 \\ 0 & 0 & -4a+29 & a-26 \end{array}\right]$$

$$r_3+r_4 \to r_4 \longrightarrow \left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 0 & 1 & a-9 & 8 \\ 0 & 0 & 4a-29 & 25 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

(*) $a-1 \neq 0 \Rightarrow$ Çözüm yok

$a=1$ ise $\left[\begin{array}{cccc} 1 & -1 & 2 & -1 \\ 0 & 1 & -8 & 8 \\ 0 & 0 & -25 & 25 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} z=-1 \\ y=8+8z=0 \\ x=-1-2z+y=1 \end{array}$ $\Big\}$ $(x,y,z)=(1,0,-1)$ tek çözüm

$\Rightarrow$ **$a=1$ ise tek çözüm, $a \neq 1$ ise çözüm yok**

2. *(25 puan)* $\left.\begin{array}{rcrcrcl} -5x_1 & + & 3x_2 & - & 2x_3 & = & 9 \\ 6x_1 & + & 2x_2 & + & 4x_3 & = & 2 \\ & & 3x_2 & + & x_3 & = & 7 \end{array}\right\}$ denklem sistemini **Cramer metodunu** kullanarak çözünüz.

$$Ax = b \Rightarrow \quad A = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 2 & 4 \\ 0 & 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix} \quad \text{ve} \quad \det(A) = -4. \ (*)$$

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 9 & 3 & -2 \\ 2 & 2 & 4 \\ 7 & 3 & 1 \end{vmatrix}}{-4} = \frac{4}{-4} = \boxed{-1}$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} -5 & 9 & -2 \\ 6 & 2 & 4 \\ 0 & 7 & 1 \end{vmatrix}}{-4} = \frac{-8}{-4} = \boxed{2}$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{\begin{vmatrix} -5 & 3 & 9 \\ 6 & 2 & 2 \\ 0 & 3 & 7 \end{vmatrix}}{-4} = \frac{-4}{-4} = \boxed{1}$$

$$|A| = \begin{vmatrix} -5 & 3 & -2 \\ 6 & 2 & 4 \\ 0 & 3 & 1 \end{vmatrix} = 0 + 3 \cdot (-1)^{3+2} \begin{vmatrix} -5 & -2 \\ 6 & 4 \end{vmatrix} + 1 \cdot (-1)^{3+3} \begin{vmatrix} -5 & 3 \\ 6 & 2 \end{vmatrix} = 24 - 28 = -4$$

3. *(25 puan)* $\begin{vmatrix} a & b & c \\ k & m & n \\ r & s & t \end{vmatrix} = 5$ olmak üzere, $\begin{vmatrix} r-5a & s-5b & t-5c \\ 5a+3r & 5b+3s & 5c+3t \\ k-4a & m-4b & n-4c \end{vmatrix} = ?$

$$\begin{vmatrix} r-5a & s-5b & t-5c \\ 5a+3r & 5b+3s & 5c+3t \\ k-4a & m-4b & n-4c \end{vmatrix} \xrightarrow{r_1+r_2 \to r_2} = \begin{vmatrix} r-5a & s-5b & t-5c \\ 4r & 4s & 4t \\ k-4a & m-4b & n-4c \end{vmatrix}$$

$$\xrightarrow{-\frac{1}{4}r_2+r_1 \to r_1} = \begin{vmatrix} -5a & -5b & -5c \\ 4r & 4s & 4t \\ k-4a & m-4b & n-4c \end{vmatrix} \xrightarrow{-\frac{4}{5}r_1+r_3 \to r_3} = \begin{vmatrix} -5a & -5b & -5c \\ 4r & 4s & 4t \\ k & m & n \end{vmatrix}$$

$$\xrightarrow{-\frac{1}{5}r_1 \to r_1} = (-5)\cdot 4 \begin{vmatrix} a & b & c \\ r & s & t \\ k & m & n \end{vmatrix}$$

$$\xrightarrow{\frac{1}{4}r_2 \to r_2}$$

$$\xrightarrow{r_2 \leftrightarrow r_3} = (-1)(-5)\cdot 4 \begin{vmatrix} a & b & c \\ k & m & n \\ r & s & t \end{vmatrix}$$

$$= (-1)(-5)\cdot 4 \cdot 5 = \boxed{100}$$

Öl: $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{vmatrix} \Rightarrow \det(A) = 5$

$a = m = 1$, $t = 5$
$b = c = k = n = r = s = 0$

$$\Rightarrow \begin{vmatrix} -5 & 0 & 5 \\ 5 & 0 & 15 \\ -4 & 1 & 0 \end{vmatrix} = 1 \cdot (-1)^{3+2} \begin{vmatrix} -5 & 5 \\ 5 & 15 \end{vmatrix}$$

$$= (-1)(-75-25)$$

$$= 100$$

NOT: ⊛ $B = [A]_{r_i \leftrightarrow r_j} \Rightarrow \det(B) = -\det(A)$ veya $\det(A) = -\det(B)$

⊛ $C = [A]_{k r_i \to r_i} \Rightarrow \det(C) = k \cdot \det(A)$ veya $\det(A) = \frac{1}{k}\det(C)$

⊛ $D = [A]_{\square r_i \to r_i} \Rightarrow \det(D) = \square \cdot \det(A)$ veya $\det(D) = \det(A)$

4. *(25 puan)* Adjoint matris yardımıyla aşağıdaki matrislerin terslerini (eğer varsa) bulunuz.

a. $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ b. $B = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

(a.) $\det(A) = 2$ (3. satıra göre kofaktör açılımı $= 1 \cdot (-1)^{3+1} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 0 + 1 \cdot (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$)

$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3$,

Benzer şekilde

$A_{12} = 1$, $A_{13} = -3$, $A_{21} = -2$, $A_{22} = 0$, $A_{23} = 2$, $A_{31} = 1$, $A_{32} = -1$, $A_{33} = 1$

$\Rightarrow \quad \text{adj } A = [A_{ij}]^T = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & -1 \\ -3 & 2 & 1 \end{bmatrix}$

$\Rightarrow \quad A^{-1} = \frac{1}{\det(A)} \cdot (\text{adj } A) = \begin{bmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -3/2 & 1 & 1/2 \end{bmatrix}$

(b.) $\det(B) = 0 \Rightarrow$ B nin tersi yoktur.

$\det(B) = 1 \cdot (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + 2 \cdot (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + 2 \cdot (-1)^{1+3} \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 0$

(1. satıra göre kofaktör açılımı)